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# Contribution to the MHD Modeling in Low Speed Radial Flux AC Machines With Air-Gaps Filled With Conductive Fluids

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This work deals with the modeling of the magnetohydrodynamic (MHD) phenomena in the air-gaps of low speed radial flux AC electrical machines filled with incompressible and electrically conductive fluids. The proposed model concerns laminar flows and it is based on a weak MHD coupling at the steady state regimes. The MHD power losses are evaluated and discussed. The model is easy to implement and could be a useful tool for the design and the optimization. An application to marine current turbine is considered.

**Index Terms**—Analytical modeling, fluid-filled air-gaps, laminar flows, low speed radial flux AC machines, MHD, power loss.

## I. INTRODUCTION

SOME specific applications use immersed air-gap AC electrical machines. These applications can be related with renewable marine energy [1], specific pumping systems [2] and air-gap reluctivity elimination by magnetic fluids [3]. The air-gaps of these electrical machines can be filled with conductive fluids (seawater...) which induces MHD phenomena due to the rotating magnetic field densities (MFDs), leading to supplementary losses and flow distortion in the air-gap. These nonconventional phenomena are often neglected in the design and the modeling procedures of such systems. It is however necessary to quantify these phenomena to check if they are insignificant, especially when high MFDs are involved, such as in superconducting electrical machines.

A rigorous modeling of the MHD phenomena may be a challenging problem leading to heavy numerical models [4][5]. These numerical models are not suitable to be used for the design, where rapid models are needed [6]. In this context, and under the assumption of laminar incompressible fluid flows, we present an analytical modeling of the MHD phenomena in the air-gaps of low speed radial flux AC electrical machines filled with electrically conductive fluids. It is based on a weak MHD coupling in the steady state regimes and assumes a sinusoidal distribution of the MFDs in the air-gap. The effect of the rotating magnetic field on the fluid flow in the air-gap is discussed and the MHD losses are evaluated. The model is easy to implement and could be a useful tool for the design and the optimization.

The study of the flow stability is out of the scope of this work. We assume that the considered systems work below the critical Taylor number determining the transition from laminar to transitional the flow of a viscous fluid between two concentric cylinders [7]–[9].

The modeled system and the MHD model equations are presented in the next section. Results and discussions are given in the third section considering an application to a marine current turbine. We don't aim any particular application, thus the dimensions of the considered machine are arbitrary chosen,

nevertheless realistic and satisfying the condition of a laminar flow.

## II. THE MHD MODEL

Using cylindrical coordinates  $(r, \theta, z)$ , we consider a sinusoidal rotating MFD oriented in the radial direction  $(B = \{B_r, 0, 0\})$  in the *mechanical air-gap* (magnetic air-gap less any nonmagnetic insulating layer thickness) of a synchronous electrical machine, filled with an incompressible and electrically conductive fluid (Fig. 1). The MFD rotates at the rotor speed  $\Omega = \omega/p$ , where  $\omega$  is the pulsation of the supply currents, and  $p$  is the number of poles pairs of the MFD.

### A. The MDH Model

We neglect the end effects; the system is thus azimuthally symmetric. The fluid flows in the air-gap in the azimuthal direction, with the angular speed  $\Omega_f(r)$  which is considered to depend only on the radial position  $r$  (laminar flow). The expression of the MFD in the reference frame of the fluid is given by (1), where  $B_m$  is the MFD magnitude and  $\theta$  is the angular position

$$B_r(r, \theta, t) = B_m \sin(p[\Omega - \Omega_f(r)]t - p\theta). \quad (1)$$

The variation of the MFD in the conductive fluid gives rise to an induced electric field  $E$  and thus an induced current density  $J$ . As described in Fig. 1, we assume that the electric field and the induced current density have only one component oriented in the  $z$ -direction ( $E = \{0, 0, E_z\}$ ,  $J = \{0, 0, J_z\}$ ) and the MFD created by the induced current is negligible compared to the MFD magnitude  $B_m$  [10]. From the Maxwell-Faraday's law, and with the notation  $(\partial_x y \equiv \partial y / \partial x)$ , we have

$$r^{-1} \partial_\theta E_z = -\partial_t B_r \rightarrow \partial E_z = -r \partial_t \theta \partial B_r. \quad (2)$$

In the steady state regime  $\partial_t \Omega_f = 0$ , thus  $\partial_t \theta = \Omega - \Omega_f$ , and (2) becomes

$$E_z = (u - r\Omega) B_r \quad (3)$$

where  $u = r\Omega_f$  is the azimuthal fluid velocity.

The induced current density in the fluid is given by (4), where  $\sigma$  is the electrical conductivity of the fluid

$$J_z = \sigma E_z = \sigma(u - r\Omega) B_r. \quad (4)$$



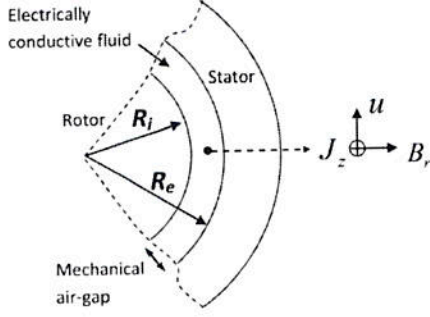


Fig. 1. The modeled system.

As consequence of the induced current, Lorentz forces are induced in the fluid, and oriented in the direction of the fluid flow ( $\theta$ ). The induced Lorentz forces in the fluid are expressed as follows:

$$F_\theta = -J_z B_r = \sigma(r\Omega - u)B_m^2 \sin^2(p[\Omega - r^{-1}u]t - p\theta). \quad (5)$$

Only the mean value  $\langle F_\theta \rangle$  of this force is considered [2]:

$$\langle F_\theta \rangle = 0.5\sigma(r\Omega - u)B_m^2. \quad (6)$$

The induced Lorentz forces affect the fluid flow. As it is a laminar flow, and due to the symmetry of the system, we consider that the fluid velocity varies only in the radial direction; thus, the variation of the azimuthal velocity  $u$  and the pressure  $P$  according to the azimuthal coordinate  $\theta$  are nil (i.e.,  $\partial_\theta u = 0$  and  $\partial_\theta P = 0$ ) in the Navier-Stokes equation [5], [7]–[9]. The governing MHD equation in the steady state regime ( $\partial_t = 0$ ) is thus reduced to (7), where  $\mu$  is the dynamic viscosity of the fluid, and with the notation  $\partial_r^2 u \equiv \partial^2 u / \partial r^2$

$$\partial_r^2 u + r^{-1} \partial_r u - r^{-2} u = -\mu^{-1} F_\theta. \quad (7)$$

We consider only the mean value of the Lorentz force (6). With  $\lambda = \sqrt{0.5\sigma B_m^2 \mu^{-1}}$ , (8) becomes

$$\partial_r^2 u + r^{-1} \partial_r u - (r^{-2} + \lambda^2) u = -\lambda^2 r \Omega. \quad (8)$$

Notice that in the case of ferrofluids, it is the magnetic forces that occur rather than the Lorentz forces [4].

The solution of (8) is given in (9), expressed in terms of the first order modified Bessel functions of the first kind  $I_1(\cdot)$ , and second kind  $K_1(\cdot)$

$$u = AI_1(\lambda r) + BK_1(\lambda r) + r\Omega. \quad (9)$$

The constants  $A$  and  $B$  are determined by the boundary conditions of the velocity on the rotor outer radius  $R_i$  and the stator inner radius  $R_e$ . The no-slip boundary conditions applies [5], [7]–[9], thus we have

$$\begin{cases} u = R_i \Omega & (r = R_i) \\ u = 0 & (r = R_e) \end{cases}. \quad (10)$$

Under these boundary conditions and with  $\alpha = \lambda R_i$  and  $\beta = \lambda R_e$ , we obtain

$$\begin{cases} A = \Omega R_e K_1(\alpha) [K_1(\alpha) I_1(\beta) + K_1(\beta) I_1(\alpha)]^{-1} \\ B = -\Omega R_e I_1(\alpha) [K_1(\alpha) I_1(\beta) + K_1(\beta) I_1(\alpha)]^{-1} \end{cases}. \quad (11)$$

Notice that when no magnetic field is applied, the fluid velocity is the homogeneous solution ( $u_0$ ) of (7) [8][9]. Considering the boundary conditions (11), and with  $\eta = R_i/R_e$ , we have

$$u_0 = \Omega(\eta^2 - 1)^{-1}(\eta^2 r - R_i^2 r^{-1}). \quad (12)$$

### B. Evaluation of the MHD Losses

The different MHD power in the air-gap are the electrical power dissipated by the Joule effect (Joule losses) in the fluid ( $P_J$ ), the electromechanical power transmitted to the fluid by the Lorentz forces ( $P_{em}$ ), and the friction power ( $P_\mu$ ) due to the fluid viscosity which creates skin friction shear stress on the rotor and stator surfaces [7]–[9]. The latter represents the hydrodynamic power transmitted to the fluid and dissipated by the fluid flow and thermal effect due to the friction between the fluid particles [9]. Since the fluid velocity is nil on the stator surface, no friction power loss occur in the latter.

The Joule losses in the fluid can be expressed as follows, where  $L$  is the axial length of the mechanical air-gap and  $E_{eff}$  is the RMS value of the electric field (the minus sign denotes a loss, and  $GV$  denotes the mechanical air-gap volume):

$$P_J = -\sigma \int_{GV} E_{eff}^2 dv = -\sigma B_m^2 \pi L \int_{R_i}^{R_e} (u - r\Omega)^2 r dr. \quad (13)$$

Considering only the mean value of the Lorentz forces, the electromechanical power is calculated as follows:

$$P_{em} = \int_{GV} \langle F_\theta \rangle u dv = -\sigma B_m^2 \pi L \int_{R_i}^{R_e} (u - r\Omega) u r dr. \quad (14)$$

As it is a post processing operation, one can solve the integrals (13) and (14) numerically.

Finally, the friction power on the rotor surface is proportional to the derivative of the velocity with respect to the radial direction [7]–[9]. It is calculated as follows, where  $I_0(\cdot)$  and  $K_0(\cdot)$  are the zero order first and second kind modified Bessel functions,  $\alpha = \lambda R_i$  and  $RS$  denotes the rotor surface:

$$\begin{aligned} P_\mu &= \int_{RS} \mu u(R_i) (\partial_r u - r^{-1} u)_{r=R_i} ds \\ &= 2\pi \mu L R_i^2 \Omega \left\{ \frac{A \lambda [I_0(\alpha) - \alpha^{-1} I_1(\alpha)] - B \lambda [K_0(\alpha) + \alpha^{-1} K_1(\alpha)]}{\alpha} \right\}. \end{aligned} \quad (15)$$

Replacing in (15)  $u$  by  $\Delta u = u - u_0$ , we obtain the variation of the friction power loss  $\Delta P_\mu$  on the rotor surface due to the application of the magnetic field, given by (16), where  $P_\mu^0$  is the friction loss when no magnetic field is applied:

$$\Delta P_\mu = P_\mu - P_\mu^0 = P_\mu - 4\pi \mu L R_i^2 \Omega^2 (\eta^2 - 1)^{-1}. \quad (16)$$



TABLE I  
PARAMETERS SPECIFICATION

Parameters	Values
Inner radius, $R_i$	1 m
Outer radius, $R_e$	1.001 m
Axial length, $L$	1.5 m
Rotor speed, $\Omega$ ( $f=50\text{Hz}$ , $p=250$ )	$0.4\pi$ Rad/s
Fluid conductivity, $\sigma$	5 S/m
Fluid density, $\rho$	$10^3$ Kg/m <sup>3</sup>
Fluid dynamic viscosity, $\mu$	$10^{-3}$ Pa.s
Taylor number, $Tn$	1579
MFD free friction loss, $P_\mu^0$	-14.9 W

It is a gain in power which is nearly equal to the electromechanical power transmitted to the fluid ( $|\Delta P_\mu| \approx |P_{em}|$ ), as long as the losses due to the friction between the fluid particles can be neglected. Indeed, the Lorentz forces tend to accelerate the fluid in the direction of the rotor motion to minimize the eddy currents, leading to a decrease of the term  $\partial_r u$  in (15), and thus a compensation of a part of the friction power on the rotor surface.

### III. APPLICATION

#### A. The System Specifications

An example of a POD PM generator for a marine current turbine is used to illustrate the MHD phenomena presented above. The chosen set of dimensions corresponds to a rough design of a 450 kW/12 rpm generator which corresponds approximately to the specifications of a 15 m diameter tidal turbine designed for a rated water velocity of 2.3 m/s. These specifications are close to those of major current industrial marine current turbine projects which have been tested at sea in the last years, as the OPENHYDRO project (<http://www.openhydro.com>) or Seagen/Seaflo projects (<http://www.seageneration.co.uk>). The chosen rated velocity is typical of high potential tidal sites located near the west European shorelines [1]. The corresponding set of parameters is presented in Table I. These parameters satisfy the condition of a laminar flow, which is verified by the evaluation of the Taylor number  $Tn = R_i(R_e - R_i)^3 \mu^{-2} \rho^2 \Omega^2$  [7], [8], where, in this case,  $R_i$  is the rotor outer radius,  $R_e$  is the stator inner radius (including the insulations thicknesses),  $\Omega$  is the rotor speed,  $\mu$  is the dynamic viscosity and  $\rho$  is the density of the fluid. It must be less than its critical value of about 1700 [8].

#### B. Results and Discussions

The MHD model is implemented in Matlab environment. The integrals (13) and (14) are evaluated numerically by using the trapezoidal numerical integration method.

The radial profile of the eddy currents amplitude is given by Fig. 2 for different values of the MFD. The mean values of the Lorentz forces have a similar distribution. The eddy currents are nil on the rotor surface where the fluid velocity is the same as that of the rotating magnetic field. The eddy currents increase linearly to reach a maximum at the stator surface where the relative velocity of the rotating magnetic field with respect to the fluid is maximal. It is not shown here, but, at high values of the MFD (case of use of superconducting materials for instance),

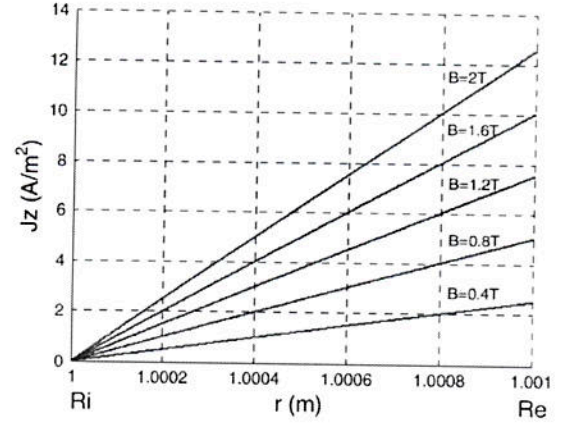


Fig. 2. Radial profile of the eddy currents in the mechanical gap for different values of the magnetic flux density.

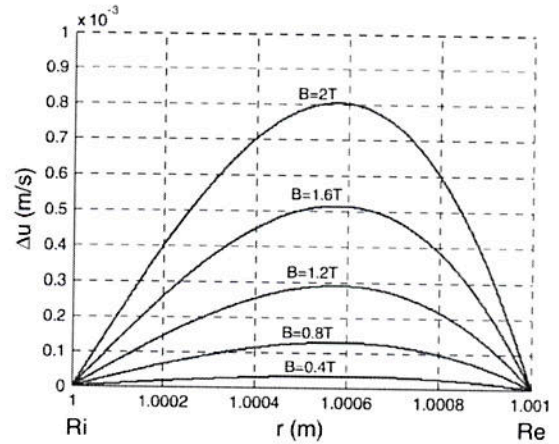


Fig. 3. Radial profile of the fluid velocity variation for different values of the magnetic flux density.

the eddy currents profile is no longer linear in the gap; since, as the MFD increases, the Lorentz forces become more important, and thus the fluid velocity tends to reach that of the rotating magnetic field over a large part of the gap starting from the rotor surface, limiting the zone of the development of the eddy currents to an area close to the stator surface.

The profile of the variation of the fluid velocity ( $\Delta u$ ) is represented in Fig. 3 for different values of the MFD. Due to the no-slip boundary condition, its maximum does not occur at the stator surface; however it is closer to the stator surface than the rotor surface, which is coherent with the variations of the eddy currents and Lorentz forces. We can also notice that  $\Delta u$  varies in a nonlinear way according to the MFD.

Fig. 4 shows the variation of the different MHD powers (gain & losses) according to the MFD. The gain in the friction power ( $\Delta P_\mu$ ) is considered positive while the Joule losses and the electromechanical power ( $P_J$  &  $P_{em}$ ) are considered negative. We can see that, as we stated above,  $P_{em}$  resulting from the numerical integration of (14) is nearly equal to  $-\Delta P_\mu$  given by (16). There is thus compensation between  $P_{em}$  and  $\Delta P_\mu$ , and therefore, the total MHD loss in the system is nearly equal to the Joule losses ( $P_J$ ) plus the MFD free friction loss ( $P_\mu^0$ ) given

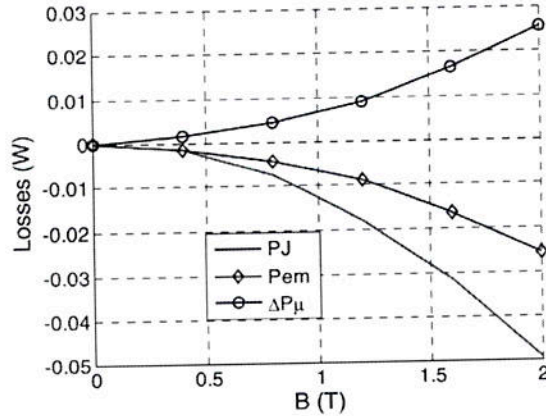


Fig. 4. MHD power losses as function of the magnetic flux density.

in Table I, which is the predominant one. The Joule losses are about twice larger than  $P_{em}$ , which is due to the weak conductivity of the fluid. For highly conductive fluids,  $P_{em}$  would be more important than  $P_J$ .

We can notice that according to the considered example, the MHD losses may be negligible compared to the machine power. However, in absolute terms, they are a part of nonconventional losses which deserve to be quantified [11].

#### IV. CONCLUSION

We have developed an analytical modeling of the MHD phenomena in the air-gaps of low speed radial flux AC synchronous machines filled with incompressible and electrically conductive fluids. The MHD power losses are evaluated and discussed. The model is easy to implement and could be a useful tool for the design and the optimization, especially when the involved MFDs are important (superconductive machines), or when the air-gap fluids are highly conductive. The model can easily be applied to the asynchronous machines by simply taking into account the slip between the rotor and the rotating MFD.

The proposed model is however limited to laminar flows and based on a weak MHD coupling. Furthermore, it assumes that the MFD distribution is sinusoidal, and only the mean values of the induced Lorentz forces are considered. Further investigations are thus needed to consider the possibility of extending the model to transitional and turbulent flows. It would be also interesting to take into account the effect of the MFD harmonics, and to study the effect of the Lorentz forces and their fluctuation on the stability of the fluids flows; this can affect considerably the hydrodynamic losses which are known to be more important in transitional and turbulent regimes.

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